K23P 1411

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)
Examination, October 2023
(2020 Admission Onwards)
MATHEMATICS
MAT3C14 : Advanced Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Distinguish between pointwise boundedness and uniform boundedness of sequence of functions on a set E.

2. Define the limit function of sequence $\{f_n\}$ of functions and show that for $m, n = 1, 2, 3, \ldots$, if $S_{m,n} = \frac{m}{m+n}$, then $\lim_{n\to\infty} \lim_{m\to\infty} S_{m,n} \neq \lim_{m\to\infty} \lim_{n\to\infty} S_{m,n}$.

3. Define beta function.

4. Show that the functional equation $\Gamma(x + 1) = x\Gamma(x)$ holds if $0 < x < \infty$.

5. Prove that a linear operator A on a finite-dimensional vector space X is one-to-one if and only if the range of A is all of X.

6. State the implicit function theorem. **(4×4=16)**

PART – B

Answer **4** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. State and prove the Stone-Weierstrass theorem.

8. a) Show that there exists a real continuous function on the real line which is nowhere differentiable.

   b) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E, then show that the $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in E$.

P.T.O.

9. a) If $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, then prove that $\{f_n + g_n\}$ converges uniformly on E.

   b) If $\{f_n\}$ and $\{g_n\}$ are sequences of bounded functions, then prove that $\{f_n \cdot g_n\}$ converges uniformly on E.

   c) Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $|f_n(x)| \leq M_n$ for $x \in E$ and n = 1, 2, 3, ..., then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

Unit – II

10. a) Suppose that the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$, and if $f(x) = \sum_{n=0}^{\infty} c_n x^n$, then prove that the function f is continuous and differentiable in (–R, R), and $f'(x) = \sum_{n=1}^{\infty} nc_n x^{n-1}$ where $|x| < R$.

    b) State and prove Taylor's theorem.

11. State and prove Parseval's theorem.

12. a) If x > 0 and y > 0, then show that $\int_0^1 t^{x-1}(1-t)^{y-1}\, dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

    b) If f is continuous (with period 2π) and if $\varepsilon > 0$, then prove that there is a trigonometric polynomial P such that $|P(x) - f(x)| < \varepsilon$ for all real x.

Unit – III

13. a) Define dimension of a vector space.

    b) Let r be a positive integer, if a vector space is spanned by a set of r vectors, then prove that $\dim X \leq r$.

    c) Show that $\dim \mathbb{R}^n = n$.

14. a) Define a continuously differentiable mapping.

    b) Suppose f maps an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$. Then prove that $f \in \mathscr{C}'(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq i \leq m$, $1 \leq j \leq m$.

15. State and prove inverse function theorem. **(4×16=64)**